ENTRAINMENT OF FERROMAGNETIC SUSPENSION BY A ROTATING FIELD

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We examined the behavior of a nonconducting ferromagnetic suspension in a uniform rotating magnetic field. The field rotation causes rotation of the suspended particles, which have intrinsic magnetic moments. Each particle becomes the center of a microscopic eddy, which all combine to generate the macroscopic (that is, hydrodynamic) motion of the medium. The equations of motion of a fluid with internal rotation are used to describe this "pumping" of angular momentum from the latent (microeddy) form into the visible form. The solution of these equations is obtained for the case in which the ferromagnetic suspension fills a cylindrical cavity and the magnetic field rotates in the plane perpendicular to the axis of the cylinder. The absorbed power and the friction torque acting on the walls of the cylinder are calculated. The computational results are compared with the available experimental data.

Entrainment of a nonconducting fluid by a rotating magnetic field was first observed in 1939 by Zwetkoff [1]. In the experiment he used an anisotropic fluid (n-azoxyanisole), whose molecules have diamagnetic anisotropy and tend to arrange themselves so that the greatest length of the molecules is parallel to the field intensity. In this case rotation of the magnetic field leads to the appearance of internal angular momentum associated with rotation of the molecules. The internal friction processes accompanying the rotation of the molecules provide for partial conversion of the internal angular momentum into the visible (hydrodynamic) form. In the case of steady rotation of the field an equilibrium distribution of the angular momentum is established between the latent and visible motions, i.e., the entire fluid is entrained into rotation.

A rotational effect analogous to the Zwetkoff effect, but far more strongly manifested, was recently observed by Moskowitz and Rosenzweig [2] in experiments with a ferromagnetic fluid. The latter was a colloidal suspension of ferromagnetic particles suspended in a nonconducting liquid.

In both of these studies an identical explanation is given for the observed entrainment of the fluid; it appears to us that this explanation is in error. In the case of steady rotation of the suspended particles the moment of the external force acting on them from the field is balanced by the moment of the viscous friction forces. The authors of [1, 2] assume that this is sufficient to entrain the fluid into the macroscopic motion. However, let us consider a suspension in which all the suspended particles rotate with the same angular velocity ω. Each particle draws into rotation the viscous fluid layers lying nearby, forming a microscopic eddy whose dimensions do not exceed the distance between the particles; in so doing each particle rotates the fluid about its own axis, therefore when averaged the microscopic eddies do not lead to any resultant hydrodynamic motion. A "macroscopic" eddy with velocity $\Omega = {}^1/_2 \operatorname{rot} \mathbf{v}$ is possible, as shown below, only in the case of nonuniform internal rotation.

1. For the theoretical description of the experimentally discovered [2] entrainment of fluid by a rotating magnetic field it is necessary to account correctly for the interaction between latent rotation (of the microeddy) and visible rotation (with the hydrodynamic velocity $\mathbf{v}$). The equations of motion of an incompressible fluid having internal angular momentum have the form [3]

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v}\right] = -\nabla p + \eta_* \Delta \mathbf{v} + \frac{1}{2\tau_s}\operatorname{rot}\mathbf{s}$$
$$\frac{\partial \mathbf{s}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{s} = -\frac{1}{\tau_s}(\mathbf{s} - J\Omega) + D\Delta\mathbf{s} + \mathbf{K}\cdot \quad \operatorname{div}\mathbf{v} = 0 \qquad \left(\eta_* = \eta + \frac{J}{4\tau_s}\right) \tag{1.1}$$

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Here $\mathbf{s}$, τ_s, and D are the volumetric density, relaxation time, and diffusivity of the internal angular momentum, J is the moment of inertia per unit volume, equal in order of magnitude to ρl^2, where ρ is the density of the suspension, l is the average distance between the suspended particles – the centers of the microeddies, K is the volumetric density of the moment of the external forces acting on the particles.

It should be noted that $\mathbf{s}$ is not simply the sum of the angular momenta $\mathbf{s}_0$ of the solid particles. Each particle draws into rotation the surrounding fluid, so that $\mathbf{s}$ is larger than $\mathbf{s}_0$, approaching the latter near the boundary of the cavity, where the fluid motion is restricted.

The system of equations (1.1) is not closed; it must be supplemented by the equations defining the external force moment density. However, such equations cannot be obtained in general form. In fact, the moment $\mathbf{k}$ of the external force acting on a single particle depends significantly on its internal state and orientation. Therefore to obtain the macroscopic volumetric density $\mathbf{K}$ we must average the microscopic moments $\mathbf{k}$, which requires special analysis in each concrete case.

We consider a ferromagnetic suspension consisting of spherical single-domain particles (if the particles are not single-domain, the results obtained below are valid in fields whose intensity is less than the coercive force, and then the residual magnetization is to be taken as m), each of which has the magnetic moment m. If the intensity H of the homogeneous rotating field is small in comparison with βM_i (β is the anisotropy constant, M_i is the specific magnetization of the solid ferromagnetic), the torque acting on an individual particle is $\mathbf{k} = \mathbf{m} \times \mathbf{H}$. In the considered case of a strongly anisotropic ferromagnetic, determination of the external torque volume density reduces to simple summation of the vectors $\mathbf{k}$, which yields

$$\mathbf{K} = (\Sigma \mathbf{m}) \times \mathbf{H} \equiv \mathbf{M} \times \mathbf{H} \tag{1.2}$$

where $\mathbf{M}$ is the specific magnetization of the fluid. Thus the system (1.1) must be supplemented by the equation describing the dynamics of the magnetization of the ferromagnetic suspension.

The magnetic moment of an individual particle is acted upon by the field $\mathbf{H}$, created by the external sources, and the local field owing to dipole – dipole interaction of the magnetic moments. However, the latter, as in a paramagnetic, has no orienting effect and therefore is not considered in the following. The equilibrium distribution of the particles with regard to the orientations of their magnetic moments in a constant field is defined by the Einstein-Fokker equation, whose steady-state solution in the present case is known [4] to coincide with the Gibbs distribution

$$w = C \exp(\mathbf{mH} / kT)$$

This implies that the system of magnetic moments behaves like a conventional paramagnetic, but with anomalously large moments of the individual particles. The equilibrium magnetization of such a "superparamagnetic" is described by the Langevin function

$$M = M_0 L\left(\frac{\mathbf{mH}}{kT}\right)$$

The magnetic moments of the particles are large; therefore saturation at room temperature is reached even in very weak fields. Thus, in a constant field we have $\mathbf{M} = (M_0/H)\,\mathbf{H}$. In a rotating field the magnetic moment approaches the instantaneous equilibrium value, which makes it possible to write the magnetization equation in the form (this equation is valid for field rotation frequencies which are small in comparison with the ferromagnetic resonance frequency)

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{1}{\tau}\left(\mathbf{M} - \frac{M_0}{H}\mathbf{H}\right) \tag{1.3}$$

where τ is the magnetization relaxation time. Since in the highly anisotropic ferromagnetic in question here the magnetic moment is rigidly coupled with the axis of easiest magnetization, rotation of the magnetic moments is possible only as rotation of the particle themselves. Therefore the time τ is determined only by the temperature, fluid viscosity, and particle dimensions.

Equations (1.1) – (1.3) form a complete system for the problem in question.

2. Let a ferromagnetic fluid fill a cylindrical cavity of radius R. The magnetic field, perpendicular to the axis of the cylinder, rotates with the constant angular velocity ω

$$H_x = H \cos \omega t,\ H_y = H \sin \omega t$$

We seek the steady-state solution of the equations of motion, in which the fluid velocity has only the φ component: $v_\varphi = v(r)$, $v_r = v_z = 0$. We see from (1.1.1) that such motion is possible if s is independent of time and $s_r = s_\varphi = 0$, $s_z = s(r)$. It follows from (1.1.2) and (1.2), in turn, that the steady solution of this form exists only if $K_z = M_x H_y - M_y H_x = \text{const}(t)$. This condition is satisfied when the magnetization vector rotates with the same angular velocity as the field, forming with the latter the constant trail angle α

$$M_x = M\cos(\omega t - \alpha),\ M_y = M\sin(\omega t - \alpha)$$

The magnetization amplitude M and the angle α are found from (1.3)

$$\operatorname{tg}\alpha = \omega\tau_s \qquad M = M_0\cos\alpha = \frac{M_0}{\sqrt{1+\omega^2\tau^2}} \tag{2.1}$$

Hence we see that the trail angle $\alpha < {}^1/_2\,\pi$ for any field rotation frequencies, and the magnetic moment volume density decreases monotonically with increase of the frequency (liquid polar dielectrics in an electric field have similar frequency dispersion [5]). This means a spreading of the probability distribution for the orientation of the magnetic moments of the individual particles as the field frequency is increased. In this process the particles themselves, in spite of the assumptions made in [1, 2], do not rotate in a stationary fashion. For stationary rotation of an individual particle it is necessary that the trail angle formed by its magnetic moment with the field direction be constant. However the Brownian motion, which varies continuously both the particle orientation and the local fields acting on the particles, prevents this constancy.

Let us turn to the definition of v and s. Excluding the pressure from (1.1.1), we obtain for the non-zero components of the vorticity $\Omega_z = \Omega$ and internal moment $s_z = s$ the equations

$$\left[r\left(\Omega - \frac{s}{4\tau_s\eta_*}\right)'\right]' = 0, \qquad \frac{D}{r}(rs')' - \frac{1}{\tau_s}(s - J\Omega) = -MH\sin\alpha \tag{2.2}$$

(prime denotes differentiation with respect to r), with the boundary conditions at the surface of the cylinder

$$v(R) = 0,\ s(R) = s_0$$

We integrate the first equation (2.2)

$$\Omega = \frac{1}{2r}(rv)' = \frac{s}{4\tau_s\eta_*} + C_1 \tag{2.3}$$

and exclude Ω from the second equation

$$s'' + \frac{s'}{r} - \varkappa^2 s = -A, \qquad \varkappa^2 \equiv \frac{\eta}{\eta_* D\tau_s}, \quad A \equiv \frac{1}{D}\left(MH\sin\alpha + \frac{J}{\tau_s}C_1\right) \tag{2.4}$$

Hence

$$s(r) = A\varkappa^{-2} + C_2 I_0(\varkappa r)$$

Here I_0 is the Bessel function of imaginary argument. The fluid velocity is now found from (2.3). After satisfying the boundary conditions we obtain

$$\begin{gathered} s(r) = s_0 + (\tau_s MH\sin\alpha - s_0)\frac{\eta_*}{\eta(R)}\left[1 - \frac{I_0(\varkappa r)}{I_0(\varkappa R)}\right], \quad v(r) = v_0\left[\frac{r}{R} - \frac{I_1(\varkappa r)}{I_1(\varkappa R)}\right] \\ v_0 \equiv \frac{R(\tau_s MH\sin\alpha - s_0)}{4\eta(R)}\,\frac{2I_1(\varkappa R)}{\varkappa R I_0(\varkappa R)}, \quad \eta(R) \equiv \eta + \frac{J}{4\tau_s}\left[1 - \frac{2I_1(\varkappa R)}{\varkappa R I_0(\varkappa R)}\right] \end{gathered} \tag{2.5}$$

We see from its definition that the values of the function $\eta(R)$ lie in the range $\eta_* > \eta(R) > \eta$. The nature of the solution (2.5) depends significantly on the value of R, i.e., on the ratio of the cylinder radius to the diffusion length

$$l_D = 1/\varkappa \approx \sqrt{D\tau_s} \qquad (\eta_*/\eta \sim 1) \tag{2.6}$$

The basis of the internal angular momentum diffusion mechanism lies in the viscosity of the suspension, while the relaxation time is determined by the viscosity and the average distance l between the solid particles (microeddy centers). Very simple estimates of the type $D \sim \nu$ and $\tau_s \sim l^2/\nu$ lead in accordance with (2.6) to $l_D \sim l$. Comparison of the calculation results with experiment [2] yields $l_D \sim 10^{-4}$ cm, which agrees with the estimate noted above. Therefore in the following we shall take $R \gg 1$.

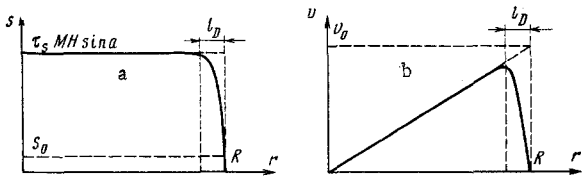

Fig. 1

The solution (2.5) for $\varkappa R \gg 1$ has a very marked boundary layer nature. Throughout the entire fluid volume, with the exception of a narrow wall layer, s is constant, equal to $\tau_s MH \sin\alpha$, and diminishes to the boundary value s_0 at distances of order l_D (Fig. 1a). The quantity s_0 characterizes the rotational intensity of only the suspended particles; its definition goes beyond the scope of hydrodynamic theory. In principle the quantity s_0 can be determined experimentally. Unfortunately, the experimental data presently available are not adequate for this purpose – the single note [2] presents only a brief description of preliminary observations. However, we can assume that s_0 is small in comparison with $\tau_s MH \sin\alpha$. In fact, $s = \tau_s MH \sin\alpha$ is the sum of the angular momenta of the microscopic eddies whose dimensions are on the order of the distance l between the particles. Since l is large in comparison with the particle dimension

$$(s_0/s) \sim (a/l)^2 \ll 1$$

Therefore we can drop s_0 in the expression for the velocity amplitude v_0.

Accounting for (2.1), then for $\varkappa R \gg 1$ we have

$$v_0 = \frac{M_0 HR}{2\eta_*} \frac{1}{\varkappa R} \frac{\omega\tau}{1+\omega^2\tau^2} \tag{2.7}$$

The velocity profile v(r) is linear up to $r \simeq R - l_D$ (Fig. 1b). We see from (2.7) that the amplitude of the macroscopic fluid motion velocity is proportional to the magnitude of the applied field and has marked frequency dispersion. The maximal rate of entrainment of the fluid by the rotating magnetic field is reached for $\omega\tau = 1$.

The dependence of the velocity amplitude on the field intensity is presented in [2] for two values of the field cyclic rotation frequency: $f_1 = 100$ Hz and $f_2 = 1000$ Hz. For both frequencies the amplitude is a linear function of the field and the slope of the line $v_0(H)$ at the frequency f_1 is less by a factor of four than for f_2. These data are sufficient to use (2.7) to determine the magnetization relaxation time. The resulting value $\tau = 2 \cdot 10^{-4}$ sec is in agreement with an independent estimate, in accordance with which $\tau \simeq 4\pi\eta a^3/kT$ is the diffusion time for rotational Brownian motion: equating these expressions for τ yields the correct order of magnitude for $a \sim 10^{-5}$ cm [6].

3. In opaque fluids, which include the ferromagnetic suspensions, measurement of the velocity profile involves serious difficulties. Far better precision can be obtained by studying the integral characteristics of the motion: the frictional torque acting on the cylinder walls and the power absorbed, which can be measured by standard electronic methods. In this connection there is interest in calculating the frictional torque M_f and the dissipative function F.

The friction torque acting per unit cylinder length is [7]

$$M_f = -2\pi R^2 \sigma_{r\varphi}(R), \quad \sigma_{r\varphi}(r) = \eta_* (v' - v/r)$$

Substituting the fluid velocity from (2.5) into the expression for the $(r\varphi)$ component of the viscous stress tensor, we obtain

$$M_f = \pi R^2 MH \sin\alpha \frac{\eta_*}{\eta(R)} \left[1 - \frac{2I_1(\varkappa R)}{\varkappa R I_0(\varkappa R)}\right]$$

Hence for $\varkappa R \gg 1$

$$M_f = \pi R^2 M_0 H \frac{\omega\tau}{1+\omega^2\tau^2} \tag{3.1}$$

Let us estimate the magnitude of the frictional torque for $\omega\tau \approx 1$. A ferromagnetic suspension with $M_0 = 60$ Gauss was used in the experiment [2]. For such a fluid with cylinder radius $R \sim 1$ cm we obtain $M_f \sim H \cdot 10^2$ dynes.

The dissipative function for a fluid with internal rotation has the form [3]

$$F \equiv F_\eta + F_\tau + F_D\,, \quad F_\eta = \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2, \qquad F_\tau = \frac{(s - J\Omega)^2}{2J\tau_s}, \qquad F_D = \frac{D}{J}\left(\frac{\partial s}{\partial x_i}\right)^2 \tag{3.2}$$

The energy dissipated per unit time is $|E^{\cdot}| = 2\int F dV$; here the integration is performed over the cylinder volume V. The primary contribution to the dissipation is that of F_τ, associated with relaxation of the internal rotation. In fact, F_η and F_D are small because of the smallness of the derivatives v' and s' through the volume of the cylinder except for the narrow wall layer of thickness l_D (Fig. 1). In this layer F_η and F_D are comparable in magnitude with F_τ, so that their contribution to the integral $E^{\cdot}$ is small in comparison with the contribution of F_τ in the ratio

$$l_D / R \equiv (\varkappa R)^{-1} \ll 1$$

For the same reason, in the expression for F_τ we can neglect $J\Omega$ in comparison with s. Thus, the dissipative function is practically constant over the entire cylinder volume and equal to

$$F \approx F_\tau \approx \frac{s^2}{2J\tau_s} \approx \frac{1}{2J\tau_s}(\tau_s M H \sin\alpha)^2$$

Thus the absorbed power is

$$|E^{\cdot}| = \frac{\tau_s}{J} M_0^2 H^2 V \frac{\omega^2\tau^2}{(1+\omega^2\tau^2)^2} \tag{3.3}$$

Maximal absorption takes place for $\omega\tau \approx 1$. In this case, using the simple estimates $J \sim \rho l^2$ and $\tau_s \sim l^2/\nu$ presented above for $M_0 = 60$ Gauss, we obtain

$$|E^{\cdot}|/V \sim 10^{-2} H^2 \ \text{W/cm}^3, \tag{3.4}$$

where H is expressed in Oersteds. Such a large value of the dissipated power makes it practically impossible to realize the ferrohydrodynamic pumps proposed in [2], based on entraining fluid by a rotating field.

4. In conclusion we shall discuss the limits of applicability of the above analysis. Equations (1.1) are linear in s. This means that they do not describe the correlation of microeddies at different spatial points. This approximation is acceptable as long as the radius of the microeddy which forms around the solid particle is small in comparison with the distance l between particles, or at least does not exceed l. However, increase of s with increase of H ($s \approx \tau_s$ MH sin α) means an increase of the radii of the "bound" eddies. Therefore, beginning with some values of the magnetic field intensity interaction of the eddies becomes significant and the linear equations will no longer be valid. In strong fields an entire spectrum of "microeddy" scales develops, which corresponds to highly anisotropic developed turbulence. There is no basis to expect that the linear dependence of the average velocity on the field intensity is maintained in the turbulent region. Such a linear dependence was observed in the experiments of [2], which confirms the applicability of our analysis for the field range used in [2].

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